**Miscellaneous Questions**

**Type – 1**

. **Comprehension-Type Questions**

• 2m 3 n 5 p is a divisor of 210.3 8.57 if m, n and p are all whole numbers such that 0 ≤ m ≤ 10, 0 ≤ n ≤ 8 and 0 ≤ p ≤ 7. Also,

(20 + 21 + 22+... + 210)(30 + 31 + 32+...+38)(50 + 51 + 52+...+ 57) = sum of all the divisors of 210.38. 57.

Q 1. The number of proper divisors of 16200 is

(a) 60 (b) 24 (c) 58 (d) 22

Q 2. The sum of proper divisors of 360 is

(a) 1170 (b) 810 (c) 1169 (d) 809

Q 3. The sum of odd divisors (≠ 1) of 10800 is

(a) 1239 (b) 1240 (c) 1238 (d) 2479

• We know that the number of positive integral solutions of the equation x1 + x2 + x3 + ... + xm = n(n ∈ N) is equal to the coefficient of xn in the expansion of (x + x2 + x3 +... to °°)m when |x| < 1. Also, we have the expansion

(1 - x)-n = n-1C0 + nC1 x +n+1C2 x2 +...+ n+rCr+1xr+1 + ... to ∞,

where |x| < 1.

Q 4. The number of positive integral solutions of the equation x1 + x2 + x3 + x4 = 10 is

(a) 9C6 (b) 10C4 (c) 9C4 (d) 10C5

Q 5. The number of negative integral solutions of the equation x1 + x2 + x3 + x4 + x5 + 10 = 0 is

(a) 10C4 (b) 10C5 (c) 9C5 (d) 9C6

Q 6. The number of ways in which 10 identical things can be distributed among 3 persons so that each gets at least one thing is

(a) 8C3 (b) 10C4 (c) 9C3 (d) 9C2

• r is called the rank of the matrix A if there exists at least one nonzero minor of order r and every minor of order r + 1 of the matrix equals zero. Again if A is a square matrix and I is the unit matrix of the same order then the equation |A - xI| = 0 is called the characteristic equation and the roots of the characteristic equation are called eigenvalues of the matrix A.

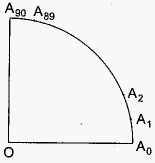
Let

Q 7. The rank of A is

(a) 3 (b) 2 (c) 1 (d) none of these

Q 8. The characteristic equation of A is

(a) x3 - 18x2 + 45x = 0 (b) x3 - 6x2 + 9x - 4 = 0 (c) x3 +18x2 - 45x = 0 (d) x3 + 6x2 - 9x + 4 = 0 Q 9. One of the eigenvalues of A is

(a) 1 (b) 2 (c) 3 (d) 5

• If α1, α2, α3, ..., αn are in an AP whose common difference is λ then we have



A quadrant 40 A90 of a unit circle is divided into

90 equal parts by inserting the points A1, A2, A3, ..., A89 on the arc of the circle. Let

cot 0.5° = a.

Q 10.  is equal to

(a)  (b)  (c)  (d) 

Q 11. is equal to

(a)  (b)  (c)  (d) 

Q 12. is equal to

(a)  (b)  (c)  (d) 

• We know from Gregory's series that

θ = tanθ – tan3 θ +  tan5θ – ... to ∞, where |θ| <

Q 13. The sum of the series to ∞ is

(a)  (b)  (c) 1 (d) 

Q 14. The sum of the series to ∞ is

(a)  (b)  (c) 1 (d) 

Q 15. The sum of the series to ∞ is

(a)  (b)  (c)  (d) 

• A second–degree curve inscribing the ΔABC, where the dies AB, BC and CA have the equations L1 = 0, L2 = 0 and L3 = 0, respectively, has the equation L1L2 + λL2L3 + μL3L1 = 0, where L and μ are parameters. Let the equations of AB, BC and CA be y = 2x, y + 2x = 0 and x = 1, respectively .

Q 16. The circumcentre of the ΔABC is

(a)  (b)  (c)  (d) 

Q 17. The parabola circumscribing ΔABC and passing through the point (4, 4) has the focus

(a) (0, 1) (b) (4, 0) (c) (1, 0) (d) (0, 4)

Q 18. The parabola circumscribing ΔABC and passing through the point (4, 4) has the latus rectum

(a) 4 (b) 1 (c) 16 (d) 8

• The director circle of a circle is the locus of points from which tangents to the circle are at right angles. C1, C2, C3, … is a sequences of circles such that Cr + 1 is the director circle of Cr. Let the equation of C1 be x2 + y2 = 4.

Q 19. The length of a chord of C11 which touches C10 is

(a)  (b) 64 (c)  (d) 32

Q 20. The distance of the chord of contact of tangents of a point on C11 with respect to C10 from the centre of C10 is

(a)  (b)  (c) 64 (d) 32

Q 21. The locus of the poles of the tangents of C9 with respect to C10 is

(a) x2 + y2 = 2 × 322 (b) x2 + y2 = 322 (c) x2 + y2 = 642 (d) x2 + y2 = 2 × 162

• The circle drawn on the major axis of an ellipse as a diameter is the auxiliary circle of the ellipse. Let an ellipse have the equation . Let p be a point on the ellipse and the perpendicular to the major axis from the point P when produced towards P meets the auxiliary circle at Q.

Q 22. The equation of the locus of the middle point R of PQ is

(a)  (b)  (c)  (d) 

Q 23. The area enclosed by the locus of R is

(a)  (b) 2πab (c)  (d) 

Q 24. The point of intersection of the tangents at P and Q (P and Q being different point) to the corresponding curves lies on

(a) b2x2 + a2y2 + a2 y2 − 2b2 x = a2 (b) y = 0

(c) b2 x2 − a2y2 + 2b2 x = a2 − b2 (d) x =0

• Two diameters AB and CD of an ellipse are called conjugate diameters if chords parallel to one of them are bisected by the other diameter. It is found that the difference of eccentric angles of A and C is . Let the equation of the ellipse be 9x2 + 16y2 = 144.

Q 25. If A =then coordinates of C are

(a)  (b)  (c)  (d) (0, 3)

Q 26. If the equation of the diameter AB is x = y then the equation of the conjugate diameter CD will be

(a) 9x + 16y = 0 (b) x + y = 0 (c) 16x + 9y = 0 (d) 9x+l6y = 7

Q 27. The area of the quadrilateral formed by the tangents at the ends of conjugate diameters AB and CD, where A =is

(a) 96 (b) 24 (c)12 (d) 48

• The quadratic equation ax2 + bx + c = 0 will have two roots equal to zero if b = 0, c = 0. Also, roots of cx2 + bx + a = 0 are reciprocal to the roots of ax2 + bx + c = 0.

A straight line y = mx+ c will be an asymptote to the curve f(x, y) = 0 if the line touches the curve at infinity. As a result, f(x, mx + c) = 0 should have two infinite roots.

Q 28. The value of λ for which the equation λ2 - 4λ + 3 + (λ2 + λ - 2)x + 6x2 - 5x3 = 0 will have two roots equal to zero is

(a) 3 (b) -2 (c) 0 (d) 1

Q 29. The value of (λ, μ) for which (λ - μ)x3 - (λ - 2)x2 - 3x + 7 = 0 will have two infinite roots is

(a) (0, 0) (b) (2,2) (c) (1, 1) (d) (2, 0)

Q 30. The equations of asymptotes to the curve x + 4xy + 3y2 + 4x - 3y + 1 = 0 are

(a)2x + 2y = 7 and x + 3y + 15 = 0 (b) x + y - 7 = 0 and x + 3y +15 = 0

(c) 2(x + y) - 7 = 0 and 2(x + 3y) + 15 = 0 (d) x + y = 7 and 2(x + 3y) + 15 = 0

• Perpendiculars AP, AQ and AR are drawn to the x-, y- and z-axes, respectively, from the point A(1, -1, 2).

Q 31. The AM of AP2, AQ2 and AR2 is

(a) 4 (b) 5 (c) 3 (d) 2

Q 32. The equation of the plane PQR is

(a) 2x + 2y - z = 2 (b) 2x - 2y + z = 2 (c) 2x - y + z = 2 (d) x - 2y + z = 2

Q 33. The volume of the tetrahedron (A, PQR) is

(a) 4 (b)  (c)  (d) 

• A ray of light emancipating from the point source P(1, -3,2) and travelling parallel to the lines

is incident on the plane x + y - 3z = 0 at the point Q. After reflecting from the plane the ray travels along the line QR. It is also known that the incident ray, reflected ray and the normal to the plane at the point of incidence are in the same plane.

Q 34. The point Q is

(a) (3,15,6) (b) (3,6,3) (c) (-3,-6,-3) (d) (-3,-15,-6)

Q 35. The equations of the line QR are

(a)  (b) 

(c)  (d) 

Q 36. The equation of the plane PQR is

(a) 5x + 2y – z + 3 = 0 (b) 11x − 5y + 2z = 30 (c) 5x − y − z = 6 (d) x − y + z = 6

• The sphere whose centre = (α, β, γ) and radius = a, has the equations (x − α)2 + (y – β)2 + (z – γ)2 = a2. Again, the intersection of a sphere by a plane is a circle.

Q 37. The distance of the centre of the sphere x2 + y2 + z2 – 2z – 4y = 0 from the origin is

(a) 5 (b)  (c)  (d) 

Q 38. The radius of the circle of intersection of the sphere x2 + y2 + z2 = 9 by the plane 3x + 4y + 5z = 5 is

(a)  (b) 3 (c)  (d) 

Q 39. The area of the circle on the sphere x2 + y2 + z2 = 25 whose centre is (1, 1, 1) is

(a) 13π (b) 3π (c) 22π (d) 25π

• Two one−parameter families of curves are called orthogonal trajectories of one another if each member of a family cuts all the members of the other family at right angles.

It is found that the orthogonal trajectories of the family of curves is the family of curves .

Q 40. The family if parabolas with a common vertex at the origin whose foci are on the x−axis and the directrices are parallel to the y−axis can have the equation (a being a parameter)

(a) y2 = 3ax (b) x2 = 4ay (c) y2 = 4a(x + 2) (d) (y − 1)2 = 4ax

Q 41. The orthogonal trajectory of the family of parabolas y2 = 4ax is

(a) x2 + y2 = c2 (b) x2 + 2y2 = c2 (c) 2x2 + y2 = c2 (d) y2 – x2 = c2

Q 42. The curve passing through the point (1, 2) that cuts each member of the family of parabolas y2 = 4ax orthogonally is

(a) 2x2 + y2 = 6 (b) x2 + y2 = 5 (c) x2 + 2y2 = 9 (d) y2 – x2 = 3

• If any point of the closed interval [α, β] is a sample point and any point of the closed interval [a, b] ⊆ [α, β] be a favorable point for the event E then the probability of the event E is defined by

P(E) = 

Q 43. The set of values of p ∈ R fro which the equation x2 + px + (p + 2) ≥ 0 for all x ∈ R is

(a) (−2, 1) (b) (2, +∞) (c) [−1, 2] (d) [1, 2]

Q 44. The set of values of p ∈ R for which the equations x2 + px + (p + 2) = 0 will have real roots is

(a) [2, +∞) (b) (2, +∞) (c) (−∞, 2) (d) R − (−1, 2)

Q 45. If p is chosen at random from the interval [0, 6] then the probability that the roots of the equation x2 + px + (p + 2) = 0 will be real is

(a)  (b)  (c)  (d) 

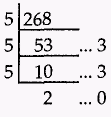
• In a series of n idependent trials for an event of constant probability p, the most probable number r of successes is given by (n + 1) p − 1 < r < (n + 1)p. Hence, the most probable number of successes is the integral part of (n + 1)p. But (n + 1)p is an integer, the chance of r successes is equal to that of r + 1 successes and both r, r + 1 are most probable numbers of successes.

A bag contains 2 white balls and 1 black ball. A ball is drawn at random and returned to the bag.

Q 46. The experiment is done 10 times. The probability that a white ball is drawn exactly 5 times is

where a0, a1, ...., an are digits whose values range from 0 to r - 1. Thus, in the scale of 7, the digits will be 0,1, 2, 3, 4, 5, 6.

**Conversion of a denary number to the scale of 5**

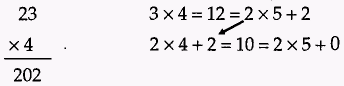
 ∴268 = 2033 in the scale of 5

**Addition of numbers in the scale of 5**

34 34 = 3 x 5 + 4

**Multiplication of numbers in the scale of 5**



On the basis of the above discussions, answer the following.

Q 55. The denary number 43125 in the scale of 6 will be represented by

(a) 353135 (b) 531353 (c) 515313 (d) 55453

Q 56. The sum of the numbers 2053 and 412 in the scale of seven is

(a) 2465 (b) 3020 (c) 3004 (d) none of these

Q 57. The product of the numbers 5623 and 6 in the scale of eight is

(a) 41672 (b) 33738 (c) 42562 (d) 45262

• Z is the set of integers and N is the set of natural numbers. If b ∈ Z, c ∈ Z and

n ∈ N are such that when b and c are divided by n, we get the same remainder then b and c are said to be congruent with respect to modulo n. Symbolically it is written as b ≡ c (mod n)

Q 58. Which of the following is true?

(a) 4 ≡ 7(mod 5) (b) 118 ≡ 18 (mod 5) (c) 110 ≡ 93 (mod 12) (d) 63 ≡ 9 (mod 11)

Q 59. Let a ≡ b (mod n), a' ≡ b' (mod n) and d, in s N- (1). Then which of the following need not be true?

(a) a + a' ≡ b + b' (mod n) (b) aa' ≡ bb' (mod n)

(c) am ≡ bm (mod n) (d) (mod n)

Q 60. If 5x ≡ 2 (mod 7) then

(a) x ≡ 6(mod 9) (b) x ≡ 6 (mod 7) (c) x ≡ 7(mod 6) (d) x ≡ 2(mod 7)

• The sum of an infinite series u1 + u2 + u3 + ... will be the finite number *l* if Sn = *l*, where Sn = u1 + u2 + u3 + ... un. In such cases it is said that the infinite series is convergent to the sum *l*.

Moreover, a series of positive terms u1 + u2 + u3 + ... up to ∞ is found to be convergent if

Q 61. The series+ to ∞ has the sum

(a) 1 (b)  (c) ∞ (d) 2

Q 62. The series(x > 0) is not convergent where x is

(a)  (b) 1 (c) 2 (d) 

Q 63. Let f(n) = . The series is convergent to the sum

(a)  (b)  (c) 1 (d) 

• The number of points of intersection of the graphs of the functions y =f(x) and y = φ(x) give the number of solutions of the equation f(x) - φ(x) = 0. Let f(x) = kex and φ(x) = x, where k is a real constant.

Q 64. The line y = x meets the curve y = kex for k ≤ 0 at

(a) no point (b) one point (c) two points (d) more than two points

Q 65. The positive value of k for which kex - x = 0 has only one real solution is

(a)  (b) 1 (c) e (d) loge2

Q 66. For k > 0, the set of values of k for which kex - x = 0 has two distinct real roots is

(a)  (b)  (c)  (d) (0, 1)

**Answers**

1c 2d 3a 4a 5c 6d 7b 8a 9c 10d

11a 12b 13b 14d 15c 16b 17c 18a 19a 20d

21c 22c 23a 24b 25c 26a 27d 28d 29b 30c

31a 32b 33c 34d 35a 36b 37b 38a 39c 40a

41c 42a 43c 44d 45d 46c 47b 48a 49b 50d

51a 52c 53a 54b 55b 56a 57c 58b 59d 60b

61a 62c 63b 64b 65a 66a